Editorial comment. Sin Hitotumatu showed that for all triangles ABC, the triangle $A'B'C'$ is acute.

Also solved by Z. Ahmed (India), A. Ali (India), A. Alt, M. Bataille (France), B. S. Burdick, M. V. Channakeshava (India), R. Chapman (U. K.), C. Curtis, N. Curwen (U. K.), P. P. Dályay (Hungary), P. De (India), A. Fanchini (Italy), D. Fleischman, O. Geupel (Germany), M. Goldenberg & M. Kaplan, J.-P. Grivaux (France), J. A. Grzesik, J. G. Heuver (Canada), S. Hitotumatu (Japan), O. Hughes, Y. J. Ionin, L. R. King, O. Kouba (Syria), W.-K. Lai & J. Risher & W. D. Ethridge, K.-W. Lau (China), J. M. Lewis, J. H. Lindsey II, G. Lord, O. P. Lossers (Netherlands), V. Mikayelyan (Armenia), J. Minkus, D. J. Moore, R. Nandan, P. Nüesch (Switzerland), C. G. Petalas (Greece), M. Sawhney, V. Schindler (Germany), M. A. Shayib, I. Sofair, N. Stanciu & T. Zvonaru (Romania), R. Stong, W. Szpunar-Lojasiewicz, H. Takeda (Japan), R. Tauraso (Italy), T. Viteam (Japan), Z. Vörös (Hungary), M. Vowe (Switzerland), T. Wiandt, L. Wimmer, J. Zacharias, L. Zhou, GCHW Problem Solving Group (U. K.), and the proposer.

A Condition for Nonexistence of Compositional Roots

11858 [2015, 801]. *Proposed by Arkady Alt, San Jose, CA.* Let D be a nonempty set and g be a function from D to D. Let n be an integer greater than 1. Consider the set X of all x in D such that $g^n(x) = x$, but $g^k(x) \neq x$ for $1 \leq k < n$. Prove that if X has exactly n elements, then there is no function f from D to D such that $f^n = g$. (Here, for $h : D \to D$, h^k denotes the k-fold composition of h with itself.)

Composite solution by Janusz Konieczny, University of Mary Washington, Fredericksburg, VA, and NSA Problems Group, Fort Meade, MD. For $h\colon\ D \to D$, let $\Gamma(h)$ denote the functional digraph of h, with an edge from a to b if and only if $h(a) = b$. From the definition of X, we see that X induces a single cycle of length n in $\Gamma(g)$. Fix x on this cycle, and suppose that f exists. Since $f^{n^2}(x) = g^n(x) = x$, vertex x lies on a cycle in $\Gamma(f)$. Let C be this cycle, and let m be its length. Both f and g permute the vertices on C; it is a single cycle under f, and g produces the nth power of this cycle.

Thus g acts on C as a product of d disjoint cycles of equal length m/d, where $d = \gcd(m, n)$. One of these cycles contains x. We have seen that the cycle in $\Gamma(g)$ containing x has length n and contains all of X. Hence g on C must produce a single cycle of length n. This requires $d = 1$ and $m = n$, which in turn requires $n = 1$.

Also solved by K. Banerjee, P. Budney, B. S. Burdick, N. Caro (Brazil), S. Chan-Aldebol, R. Chapman (U. K.), P. P. Dályay (Hungary), O. Geupel (Germany), H. B. Ghaffari (Iran), E. A. Herman, T. Horine, Y. J. Ionin, B. Karaivanov (U. S. A.) & T. S. Vassilev (Canada), K. E. Lewis (Gambia), J. H. Lindsey II, J. Olson, J. M. Pacheco & Á. Plaza (Spain), A. J. Rosenthal, A. H. Sadeghimanesh (Denmark), J. Schlosberg, J. H. Smith, R. Stong, T. Viteam (Japan), GCHQ Problem Solving Group (U. K.), TCDmath Problem Group (Ireland), and the proposer.

Avoiding Voids

11862 [2015, 802]. *Proposed by David A. Cox and Uyen Thieu, Amherst College, Amherst, MA.* For positive integers n and k, evaluate

$$\sum_{i=0}^{k}(-1)^i\binom{k}{i}\binom{kn-in}{k+1}.$$

Solution I by Borislav Karaivanov, Sigma Space, Lanham, MD, and Tzvetalin S. Vassilev, Nipissing University, North Bay, Ontario, Canada. The value is $kn^{k-1}\binom{n}{2}$.

Consider a deck of kn cards, with n distinct cards in each of k suits. Both the summation and the value count the ways to pick $k+1$ cards with at least one card from each suit. For the value, we pick one of the k suits to contribute two cards and pick one card from each of the other suits.